7. Denn shows that the wave velocity, U, is a function of x and t for viscoelastic fluids.

Answer: We agree. As I stated earlier, in my papers, U is a function of t for Newtonian fluids and a function of x and t for power-law and Maxwell fluids, since r is a function of t in draw resonance.

I sincerely hope that I have cleared all the misunderstanding about my papers and that the facts will be clear to all in this field when Parts I, II and III of my paper are read together.

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Digital Control of Time-Delay Processes

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A digital controller for time-delay processes was recently proposed by Gautam and Mutharasan (1978). This note examines the control logic in more detail and compares it with previous work.

THE PROPOSED CONTROLLER

The digital controller of Gautam and Mutharasan can be written for the process system of Figure 1 as

$$m_i = K \ (y^{set} - y_i) + \frac{y^{set}}{K_p} - \bar{L}_{i-(n+1)} \quad (1)$$

where $\bar{L}_{i-(n+1)}$ is an estimated past load, based on an assumed process model. The calculation of this load term, detailed in the cited paper, is based on:

a) Assuming a known first-order/delay model $K_p e^{-\alpha s}/(\tau s + 1)$ for the process transfer function.

b) The delay α defined in terms of the sampling period T as $\alpha = (n + \beta)T$, where n is an integer and $0 \leq \beta \leq 1$.

c) From the integrated model equation and the output data, it is possible to calculate an earlier input $x_{i-(n+1)}$.

d) Since the earlier controller output $m_{i-(n+1)}$ is known, then $\bar{L}_{i-(n+1)}$ is estimated as $x_{i-(n+1)} - m_{i-(n+1)}$. The input data for this calculation are the present and past samples of the process output, along with previously-generated data.

The first term in the controller equation above is the usual proportional-mode output error term. The gain K is an adjustable controller parameter. Although no tuning guide is given, tuning should not generally be a problem. If the time delay is relatively large, the term can introduce oscillation into the output. This is because the undelayed output is unmeasurable, and the delayed output y_i must be used in the error term.

The second term is an "ideal preload," based on the process gain K_p. This has been discussed by Shinsky (1967) and Bohl and McAvoy (1976) as an improvement over the integral mode for setpoint changes. The improved response of the proposed controller over a PID controller for setpoint changes is probably due largely to this term. If the actual process gain differs from the assumed value (see Gautam and Mutharasan, Figure 4, curve $e_\tau = 0.25$, $e_k = -0.25$), then the speed of response can be affected significantly.

The model-based third term, designed to correct for load changes and model error, is the novel part of the controller. It is evident from Figure 1 that L_i, if known and constant over the sampling period, could be subtracted from the controller output m_i to exactly compensate for the load in a feedforward manner. Since L_i is not known and cannot be estimated, the approximation proposed is to use the estimate $\bar{L}_{i-(n+1)}$ of an earlier load effect. As can be visualized in Figure 2 for the case where $\alpha = 3.3T$, there is no reason to expect the approximation $\bar{L}_{i-4}$ to be a good estimate of the current load, unless the load is constant.

The role of the third term can be better viewed as a feedback mechanism. The present output y_i is used to generate an earlier

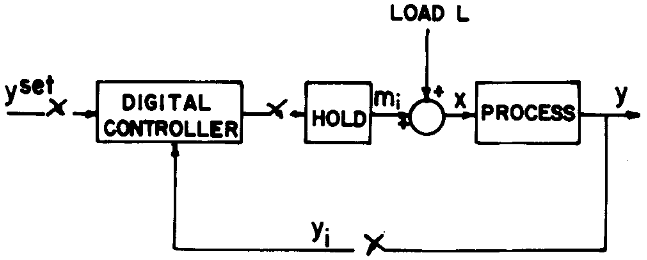

Figure 1. The controlled process system.

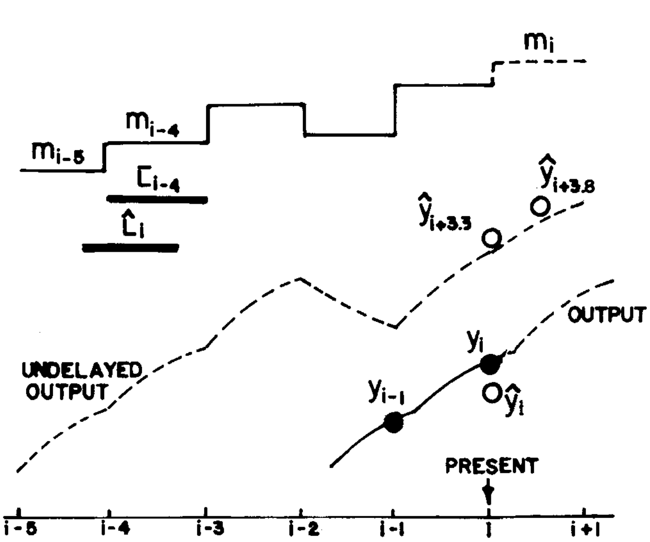

Figure 2. Variables of the two controllers for a time-delay of 3.3 units.

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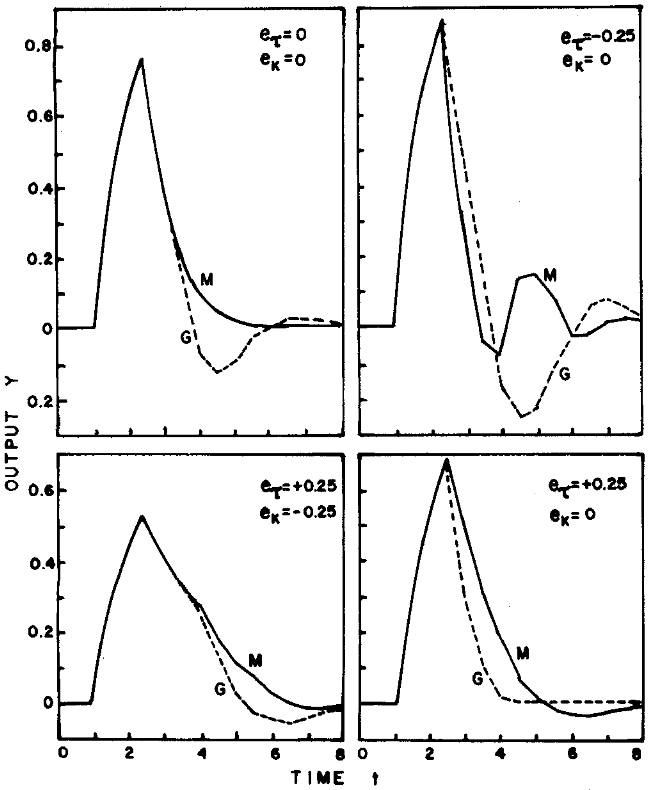

Figure 3. Response to load step change with Gautam-Mutharasan (G) and Moore et al. (M) controllers. [e_τ and e_K are the fractional errors in the process time constant and gain].

model-based process input $x_{i-(n+1)}$. If this does not equal the corresponding controller output $m_{i-(n+1)}$, then the difference $\bar{L}_{i-(n+1)}$ is incorporated into the present controller output. In this way, the third term tends to drive the error to zero, much as an integral mode does. It is a feedback mechanism, based on output samples, that can be considered as an "integral-equivalent mode" to compensate for disturbances and model error. However, the term lacks the tuning capability of the usual integral mode, and would be more flexible with an adjustable coefficient.

From this discussion, we can view the proposed controller as a form of PI control with ideal preload. The proportional-mode term is an important part of this controller for any practical process situation, rather than an optional addition. Incidentally, the work of Bohl and McAvoy suggests that an output derivative term might be a useful addition to Equation (1). This, however, would depend on the noise level, delay magnitude, and sampling rate.

A RELATED APPROACH

It is interesting to compare the proposed controller with the integral-predictor controller of Moore et al. (1970), which was discussed more recently by Meyer et al. (1978). While differing in development, this controller also includes (a) an estimated load term as an integral-equivalent mode, (b) a form of preload, and (c) the prediction of process variables using an assumed first-order/delay process model.

Following Meyer, the control law can be written as

$$m_i = K_c\ (R_i - \hat{y}_{i+\theta}) - \hat{L}_i \tag{2}$$

where R_i is a calibrated setpoint, $\hat{y}_{i+\theta}$ is a prediction of the undelayed output, and $\hat{L}_i$ is an estimate of an earlier load. The calibrated setpoint is simply a preload term to provide unity loop gain for setpoint effects and is based on loop parameters. The term $\hat{y}_{i+\theta}$ is a prediction of the undelayed process output based on the first-order/delay process model. It is assumed that the zero-order hold adds an apparent delay of $0.5T$ so that the total effective delay is $\theta = (n + \beta + 0.5)T$. The predictor first estimates $y_{i+\alpha}$ and then $y_{i+\theta}$, as shown in Figure 2 for $\alpha = 3.3T$.

The load estimate $\hat{L}_i$ is defined as

$$\hat{L}_i = \hat{L}_{i-1} + K_I T(y_i - \hat{y}_i) \tag{3}$$

where $\hat{y}_i$, as shown in Figure 2, is an estimate of y_i based on the previous sample point and the process model. The coefficient K_I is a second controller parameter. This load estimate $\hat{L}_i$ is an estimate of an earlier load that affects the present output y_i, as shown in Figure 2. As can be seen, this integral error term serves a role similar to that of the $\bar{L}_{i-(n+1)}$ term of Gautam and Mutharasan.

The integral-predictor controller can also be written (Meyer et al.) as

$$m_i = \frac{K_c}{1 + K_c K_p(1 - A)}[R_i - A\ \hat{y}_{i+n+\beta}] - \hat{L}_i \tag{4}$$

where $A = \exp(-T/2\tau)$. This form of the controller suggests that it is a PI controller for high sample rates ($A \rightarrow 1$) and acts as an ideal preload/integral feedback controller for very small sample rates ($A \rightarrow 0$).

SIMULATION STUDY

A brief study was made of the effect of model error on the performance of these two controllers. The test problem selected was that of Gautam and Mutharasan ($T = 0.5$, $K_p = 1$, $\tau = 1$, $a = 1$, unit step load change). The four cases studied (no model error, +25% error in τ, −25% error in τ, and a combined error of +25% in τ and −25% error in K_p) were the same cases presented by Gautam and Mutharasan. The adjustable controller parameter K in the Gautam-Mutharasan controller was selected as the same arbitrary value (0.5) that their study indicated for a reasonable response. The question then arose as to what values would be suitable for the two adjustable parameters (K_c and K_I) of the integral-predictor controller. Moore et al. provided model-based equations for these parameters so as to give deadbeat response. Since this obviously would be too stringent a specification in the presence of model error, we decided arbitrarily to select values that were 60% of the model-based deadbeat values. This provided controller settings of $K_c = 2.11$ and $K_I = 3.05$ for the integral-predictor controller.

Results of the simulation study are shown in Figure 3. This simply shows that the Moore controller can give results that are comparable to that of Gautam and Mutharasan for the four cases. Further tuning of either controller could give results that would better meet particular response criteria. Incidentally, a fifth case (−25% error in τ, +25% error in K_p) was also examined. While these results are not shown, responses were highly underdamped for both controller types. The Moore controller, however, was somewhat more stable for this case.

DISCUSSION

These two controllers can be classified as proportional-integral algorithms that utilize model information to overcome some of the limitations of conventional PI control for time-delay processes. The control logic in both cases reduces to relatively simple, straightforward algorithms, with very modest storage requirements.

The proportional mode of Moore et al. is novel, since the output signal is based on a prediction of the undelayed output. This is a possible way to utilize available model information, and might even be effective in the proportional mode of Gautam and Mutharasan.

Both the calibrated setpoint of Moore et al. and the preload term of Gautam and Mutharasan are designed to reduce setpoint sensitivity. The calibrated setpoint was proposed by Moore for a predictor algorithm that had no integral action. Since the integral-predictor algorithm has an integral-equivalent mode, the proportional-mode term might be more effective if y^{set} were used instead of a calibrated setpoint. The separate preload term could then be added to the controller, as was done by Gautam and Mutharasan.

The two integral-equivalent modes, while different in design, serve the same function. But the use of an adjustable integral coefficient might provide more flexibility for the Gautam and Mutharasan approach. It is difficult to assess the relative merits of the two integral approaches from the brief simulation study described here.

The features noted above suggest several new model-based controller algorithms for time-delay processes. In addition, there are other reported model-based algorithms that should be considered. These include the discrete version of the classical Smith predictor (Meyer et al.), various prototype algorithms (Chiu et al. 1973), and the algorithms of Takahashi and colleagues (Tomizuka et al. 1978, Auslander et al. 1978). The last group of algorithms is of interest because of the development of a simplified z-domain process model based on step response data.

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Fluid Pressure Distribution in an Aerated Hopper

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Aeration of hoppers is now widely used, both to increase the discharge rate of solids and as a flow promoter for non-free flowing materials. Any theoretical approach to the stress analysis or to the prediction of discharge rates from an aerated hopper needs the fluid pressure distribution in the hopper. In this note, an analytical expression for the fluid pressure distribution in an aerated hopper is developed.

THEORY

It is assumed that before gas is added to the hopper, there is zero fluid pressure gradient in the particle bed. That is, the discharge rate at gravity flow is not affected by the presence of the interstitial gas. This assumption is quite reasonable, for particles greater than about 100 μm in diameter. It should, however, be borne in mind that for fine particles, the negative gauge pressure created during the flow is appreciable, and it affects the discharge rate of solids. Thus, we conclude that the relative velocity u between the fluid and the particles is caused by the additional gas input through the walls (Papazoglou and Pyle 1970).

The gas is introduced into the hopper through a porous conical aeration section, shown in Figure 1. Gas velocity entering the bed from this conical distributor is assumed to be uniform. According to Darcy's Law, the fluid pressure gradient in a particle bed is proportional to the relative velocity between the gas and the particles (Davidson and Harrison 1963). Therefore the components of the interstitial gas velocity are given by

$$u_r = v_r - K_p \frac{\partial p}{\partial r} \qquad (1)$$

$$u_\theta = v_\theta - \frac{K_p}{r} \frac{\partial p}{\partial \theta} \qquad (2)$$

With the bulk density of the bed assumed constant (Altiner 1975), the continuity equations for the particles and the gas are (Bird et al. 1960)

for particles

$$\nabla \cdot v = \frac{\partial v_r}{\partial r} + \frac{2v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta \cot\theta}{r} = 0 \qquad (3)$$

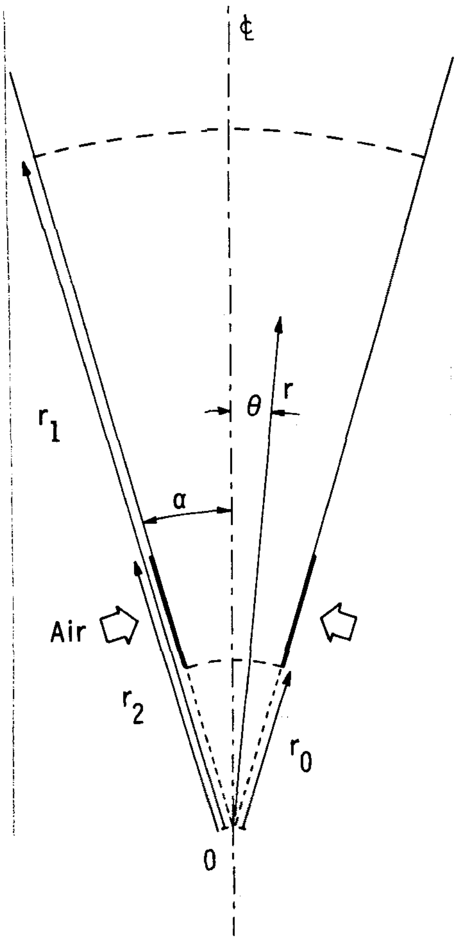

Figure 1. Hopper configuration.

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